JOURNAL OF MATERIALS SCIENCE 3 (1968) 596–602

Recovery Creep in Materials Hardened by a Second Phase

R. LAGNEBORG
AB Atomenergi, Stockholm, Sweden

Received 17 May 1968

A recovery creep model, based upon previous theories by McLean and co-workers, has been developed for creep in materials hardened by a second phase. According to the model the increased creep strength in these materials is caused by a decrease in the recovery rate, and this in turn is due to a decrease of the driving force for the recovery process and of the mobility of the climbing dislocations involved in the process. It is shown that the model can account for the very large stress-dependence of the creep-rate often found for alloys hardened by a second phase. Another support for the model is the observation that changes in the creep-rate for materials in different states of precipitation-hardening are entirely due to changes in the recovery rate.

1. Introduction

Ansell and Weertman [1] appear to have been the first to develop a quantitative theory for high-temperature creep of alloys hardened by a second phase. They assumed that the rate-controlling process is the climb of dislocations over the second-phase particles. Depending on the stress level, the details of this process may differ slightly. At low stresses they suggested that the dislocations climb over the particles with no pile-up or bowing of dislocations at the particles, and for this case they arrived at the following expression of the creep rate:

$$\dot{\epsilon} = \frac{\pi \sigma b^3 D}{2 kT d^2} \qquad (1)$$

where σ is the applied stress, b the Burgers vector, D the self-diffusion coefficient, k Boltzmann's constant, T the temperature, and d the particle size. At stresses larger than $\mu b/\lambda$, where μ is the shear modulus and λ the inter-particle spacing, the dislocations will move past particles by bowing out and pinching off loops around the particles. This occurs until the back stress exerted by the loops around the particles prevents new dislocations from bowing out between the particles. The steady-state creep rate will then be governed by the rate at which the dislocation loop nearest to the particle climbs to the top of the particle and is annihilated; the other loops will then move inwards, and a new loop will form by bowing and pinching of an arrested dislocation. This process gives an equation

$$\dot{\epsilon} = \frac{2\pi \sigma^4 \lambda^2 D}{d \mu^3 kT} \qquad (2)$$

From these expressions we would expect that the creep rate/stress relation obeys a fourth power law at high stresses, and a first power law at low stresses. This type of stress-dependence is not in agreement with experiments. In general, the stress exponent is much higher than 4 for dispersion-hardened alloys. For thoriated nickel a value of 40 has been found [2], for thoriated Ni/20% Cr values between 10 and 25 [3], and for an aluminium SAP alloy a value above 10 [4]. Ansell and Lenel [5] obtained for another SAP alloy an exponent of 4 in the high stress region, in conformity with the theory described above. However, in the low stress range below about 1.2 kg/mm^2 ($\sim 0.6 \times 10^{-3}$ μm), where the stress-dependence of the creep rate according to the theory should decrease, instead it increased, and the exponent became rapidly larger than 4. As regards precipitation-hardened alloys, e.g. the precipitation-hardenable nickel base alloys, there appear to be very few direct experimental studies on the stress-dependence of the creep rate. Reported experiments [6] yield values from about 4 to 7. The special class of creep-resistant

austenitic stainless steels usually alloyed with titanium or niobium, that are strengthened by particles precipitating during the creep process, also seem to have strongly stress-dependent creep rates. Evaluation of creep results from a 15% Cr/15% Ni/Mo/Ti/B steel [7] gives a stress exponent of about 9.

Hence, the conclusion one can draw from the experimental results for dispersion- and precipitation-hardened alloys available at present is that the stress-dependence of the creep rate may vary widely between different materials and structural conditions. There are also indications that the stress-dependence might increase at low stresses. The present theories by Weertman and Ansell are apparently not able to account for this behaviour. In the following section a model for high-temperature creep in materials hardened by a second phase will be described. Essentially, it is based upon the model for recovery creep in pure metals and solid solution alloys proposed by McLean [8] and McLean and Hale [9]. As we shall see, such a model may account for the observed stress-dependence of the creep rate and furthermore it appears, from quite general arguments, that such a recovery-controlled creep model is necessary for materials containing precipitates.

2. Recovery Creep Applied to Materials Hardened by a Second Phase

2.1. Recovery Creep in Pure Metals and Solid Solutions

The most widely accepted concept of high-temperature creep in pure metals and solid solutions seems to be the one proposed by McLean and co-workers [8-10]. In agreement with direct observations, the model assumes that the dislocations formed during creep exist in a three-dimensional network. During primary creep the dislocation density increases; this implies that the rate of recovery cannot catch up with the rate of strain-hardening, and this causes the creep-rate to decrease. As the mesh-size in the dislocation network decreases with increasing dislocation density the recovery accelerates, and eventually a stage is reached where there is a balance between strain-hardening and recovery. The creep-rate in this steady state is given by

$$\dot{\epsilon}_s = r/h \qquad (3)$$

where r is the recovery rate, $\partial\sigma/\partial t$, and h the strain-hardening, $\partial\sigma/\partial\epsilon$. This stationary creep process may be regarded as consecutive events of recovery and strain-hardening of the dislocation network. McLean and Hale assume that the junctions of the network provide the strength [9]. Some of these junctions, most probably those connected with the longest dislocation links, will break as a result of thermal fluctuations. These links will move a certain distance before being held up by the network, thus giving a strain increment but also some strain-hardening, since the total length of dislocations concomitantly increases.

The recovery which takes place is a decrease in the dislocation density, and consequently the mesh-size of the dislocation network increases. Eventually this causes some junctions to become sufficiently weak to break free, and the consecutive events of recovery and strain-hardening can repeat themselves.

2.2. General Arguments for a Recovery Creep Model in Alloys Hardened by a Second Phase

The dislocations which have climbed according to the Ansell-Weertman model for dispersion-hardened alloys [1] subsequently glide, meet dislocations of opposite sign, and annihilate. Thereby the total dislocation length has been reduced, i.e. recovery has taken place. The rate of this recovery has not been considered in detail by Ansell and Weertman.

Such a description of creep is not likely to be valid in materials hardened by a second phase, because of the following general arguments. Since it is the slowest step that determines the rate of a process consisting of sequential events, Ansell and Weertman obviously assume the recovery step to be faster than the dislocation climb over the particles. However, such a situation would, at the best, give a constant creep-rate from the very beginning of the creep test, firstly because the time for the dislocations to surmount the particles will not change as creep proceeds, secondly because the product of the number of dislocation sources and the plastic strain produced by each loop expanded according to the model is constant (cf [1] p. 842). Such behaviour, of course, contradicts all experimental experience, since alloys hardened by a second phase, like all pure metals and solid solutions, exhibit a primary creep range with decreasing creep-rate [6]. If anything, one would probably expect the product of number of sources and the strain produced by each loop

to increase in the primary stage where the dislocation density increases, and consequently this would rather make the creep rate increase. (If every link in the dislocation network is a potential source, the density of sources would be proportional to $\rho/(\rho)^{-1/2} = \rho^{3/2}$, where ρ is dislocation density, and if the area swept by the loops equals the area of the meshes in the network, which is proportional to $[1/(\rho^{1/2})]^2 = 1/\rho$, the product of these two factors is proportional to $(\rho)^{1/2}$). Hence we do not think that the Ansell-Weertman model adequately accounts for the creep process in the secondary stage. Instead, it seems as if it would be more successful in explaining the creep-rates at short times in the primary stage before the dislocation density has appreciably increased.

Now, when we have seen that a creep model, in which the strain-hardening step is slower than the recovery step and is thus rate-controlling, is unable to account for the transition from primary to secondary creep, we shall consider the situation when the recovery step is the slowest. As mentioned above, the Ansell-Weertman equations are likely to give the creep-rate in the beginning of the primary stage, since the obstacles in the form of the second-phase particles exist from the very beginning, and therefore this model can work fully effectively from the start. When recovery is slower than strain-hardening, the dislocation density will increase steadily. This implies that the density of obstacles for dislocation movement increases. Alternatively, one may say that the link length in the dislocation network decreases. Such obstacles due to interaction of dislocations can indeed be very strong; attractive junctions in dislocation networks may have a strength of several hundred eV [9]. This implies that eventually the dislocation density will be large enough for the creep-rate to be controlled by the link size, which is related to the effective strength of the junctions in the dislocation network. From that stage and onwards the creep-rate will gradually decrease as dislocation density increases. The fact that the creep-rate does not continue to decrease implies that the recovery rate increases with decreasing mesh-size of the dislocation network. Eventually a stage will be reached where the recovery rate will exactly balance the strain-hardening and the creep-rate will be constant.

As we see, such a description is completely analogous to the recovery creep model for pure metals and solid solutions [8, 9]. One might then ask what gives the precipitation-hardened alloys their creep resistance. Obviously the second-phase particles will impede the growth of the mesh-size in the dislocation network, and hence recovery and creep-rate will be slowed down. Therefore, in such a recovery creep model the important effect of the particles is that they retard recovery, which is the rate-controlling step. The details of this effect will be treated in the two following paragraphs.

2.3. The Recovery and Strain-Hardening Steps

The growth of the average mesh-size (R_m) of the dislocation network constitutes the recovery process in the present case. This process occurs essentially by climb of dislocations and simple reasoning shows that the driving force is inversely proportional to R_m [11]. The resulting growth-rate of the average mesh can be written

$$dR_m/dt = M \cdot \tau/R_m \qquad (4)$$

The factor τ/R_m, where τ is the line tension of the dislocations, is the driving force for the growth process, and the proportionality factor M can be regarded as the mobility of a climbing dislocation.

Normal grain growth is dealt with in an exactly analogous manner, and one obtains an equivalent equation for the rate of grain growth. It is well established that precipitates and inclusions retard grain growth considerably; this effect has also been given a theoretical explanation by Zener [12] and has later been considered in more detail by Hillert [13]. One assumes that the grain-boundary experiences a back stress from the particles which hold up the boundary, and therefore the driving force determined by the grain radius should be decreased by the amount of this back stress.

Because of the analogy between grain growth and growth of the meshes in a dislocation network, it seems that we can apply this treatment to the growth of a dislocation network which is impeded by second-phase particles. For the general case, the retarding force will be denoted by $\tau \cdot z$, where z depends on number and sizes of the impeding particles, and also on the exact nature of the interaction between the particle and the dislocation. At present it does not seem possible to make a realistic estimate of the magnitude of this interaction. By analogy with the treatment by Hillert [13] the growth-rate

of the average mesh size is then given by

$$\frac{dR_m}{dt} = \frac{M\tau}{2R_m}(1 - z R_m)^2 = \frac{M\tau R_m}{2}\left(\frac{1}{R_m} - z\right)^2 \quad (5)$$

It is well established, both experimentally and theoretically, that the flow stress at low temperatures is proportional to the square root of the dislocation density, ρ [14]. The same type of relation has been verified for iron strained under creep conditions [9]. By the aid of this relation between creep stress and dislocation density in the secondary stage, equation 5, and the relation $R_m = (\rho)^{-1/2}$ between average mesh size, R_m, and dislocation density, we can now deduce an expression for the recovery rate $r = \partial\sigma/\partial t$.

$$r = \frac{M\tau}{2}\sigma\left(\frac{\sigma}{\alpha\mu b} - z\right)^2 \quad (6)$$

Beside the effect of second-phase particles on the driving force for the recovery of the dislocation network, we should also expect an effect upon the mobility, M, of the climbing dislocations. Since the dislocations will be held up at the particles, the overall mobility will be determined by the local mobility at the particles. For example, if the dislocations at least partially climb inside the particles, a decrease of the mobility could be due to slower self-diffusion in the second-phase particles; the large observed activation energies for the creep-rate in SAP, TD nickel, and precipitation-hardened nickel alloys have been explained in this manner [15]. If the dislocation has to climb around the particle, as is assumed in the Ansell-Weertman theory, the overall mobility will be governed by the time it takes to climb to the top of the particles, and hence mobility will in this case be inversely proportional to the particle-size d. From electron-micrographic observations in an Ni/Cr/Al alloy, Gibbons [16] concluded that dislocations both climb around and shear off precipitate particles. In an actual case the mobility of climbing dislocations may therefore be decreased by both mechanisms described above. However, irrespective of mechanism, it is likely that the applied stress enhances the climb rate by a small influence on the activation energy. If, following McLean [8], we assume that this effect increases linearly with stress, the recovery can be written as

$$r \propto \sigma^2\left(\frac{\sigma}{\alpha\mu b} - z\right)^2 \quad (7)$$

Except for the impeding influence from the second-phase particles on the driving force and the mobility, the treatment above is identical with that by McLean [8] for pure metals and solid solutions.

The fact that recovery was slower than strain-hardening means that, although the slip of a dislocation link that has broken free will be slowed down by climb over intervening particles, this process will be faster than the recovery. This also means that the strain increment, $\partial\epsilon$, for a certain decrease in internal stress by recovery, $\partial\sigma$, will only be limited by the density of dislocation obstacles, i.e. dislocation density. Hence, the strain-hardening $\partial\sigma/\partial\epsilon$ during creep will be essentially the same in a material hardened by a second phase as in a pure metal or solid solution. At ambient temperatures the stress-strain relation for plastic deformation often obeys a parabolic law; this gives a strain-hardening that is inversely proportional to the stress. Determinations of the strain-hardening on creep-tested material, which are very difficult to perform, indicate that strain-hardening may be somewhat more strongly stress-dependent, for nickel, $h \propto \sigma^{-1.7}$, and for aluminium, $h \propto \sigma^{-1.1}$ [10]. If, for the sake of simplicity, we here assume that the rate of strain-hardening is inversely proportional to the stress, we get the following expression for the creep-rate:

$$\dot{\epsilon} \propto \sigma^3\left(\frac{\sigma}{\alpha\mu b} - z\right)^2 \quad (8)$$

2.4. Comparison with Experimental Results

According to the discussion in the previous section, changes in creep-rate for materials with varying states of hardening, e.g. different volume-fractions of second-phase particles, should entirely be due to changes in recovery rate. Recent creep and recovery experiments by Gibbons [16] on precipitation-hardened Ni/Cr/Al alloys at constant stress show that there is a proportional relation between creep-rate and recovery rate for materials with different content and dispersion of the γ' precipitate. This result strongly supports the recovery creep model for materials hardened by second-phase particles.

Fig. 1 shows measurements of the dislocation density for steady-state creep at 700° C in an austenitic 20% Cr/35% Ni stainless steel containing γ' precipitates [17], and the data points fall approximately on a straight line in a σ versus $\sqrt{\rho}$ graph as was assumed in the previous

section. However, the σ versus $\sqrt{\rho}$ relation exhibits a non-zero stress intercept and the complete relation may be written as

$$\sigma = \sigma_0 + \alpha\mu b\sqrt{\rho} \qquad (9)$$

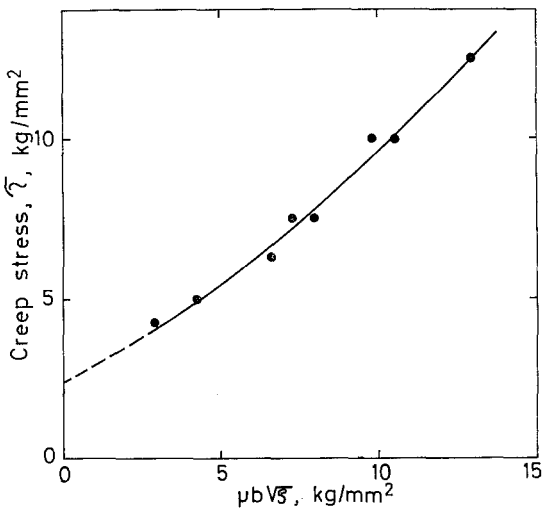

Figure 1 Creep stress versus $\mu\, b\sqrt{\rho}$ for a 20% Cr/35%/0.5% Ti/0.5% Al alloy creep-tested at 700° C. μ is shear modulus, b Burgers vector, and ρ dislocation density in the secondary stage. τ is the shear stress and equals half the tensile stress. Each point in the graph represents a count of the number of dislocations over a length of about 0.1 cm [17]

where σ_0 is the stress intercept, α is a constant, μ the shear modulus, and b the Burgers vector. Apparently σ_0 is related to the impeding effect of the γ' precipitates during creep. In fact it appears that σ_0 is directly connected with z, which was a measure of the retarding force from the second phase on the climbing dislocation during recovery. If we employ the original equation for the recovery rate (equation 4) and the actual relation between stress and dislocation density (equation 9) and go through the same treatment as in the previous section, we arrive at the following expression for the creep-rate:

$$\dot{\epsilon} \propto \sigma^2 \left(\frac{\sigma}{\alpha\mu b} - \frac{\sigma_0}{\alpha\mu b}\right)^3 \qquad (10)$$

This expression, except for the interchange of the exponents between the two factors, is analogous to equation 8. From this it follows that we can expect z to be approximately equal to $\sigma_0/\alpha\mu b$.

To illustrate the effect of second-phase particles, equation 8 has been plotted schematically in fig. 2 for assumed values of the proportionality constant and z. If the second-phase particles only decreased the driving force for the recovery process, the stress versus creep-rate curve would be shifted from curve A to B, if A corresponds to a material free of particles. The decrease in mobility of climbing dislocations due to the presence of second-phase particles will further retard the creep-rate. The proportionality constant in equation 8 will thereby be lowered by a certain constant factor and this results in a final stress/creep-rate curve C at still lower creep-rates and running parallel with curve B. Evidently the decrease in the driving force for the recovery causes the creep-rate to become gradually more stress-dependent. Fig. 2 shows that for stresses just above the critical, which is the stress that gives zero creep-rate according to equation 8, the exponents in the usual power law, $\dot{\epsilon} \propto \sigma^n$, may decrease from 11 to 5 for an increase of the stress by a factor of 4. Hence, depending on the stress range, the stress sensitivity of the creep-rate for a particular material might vary considerably.

It is suggested that this is the explanation of the considerable scatter in the observed stress exponents for dispersion- and precipitation-hardened alloys. Furthermore, it is clear that equation 8 is able to account for the observed increase of the stress sensitivity of the creep-rate with decreasing stress in SAP [5]. One might ask why such an increase in stress sensitivity has not been observed more frequently. One obvious reason is that rather few accurate measurements of the stress/creep-rate relation exist for materials hardened by a second phase, and, moreover, the measurements made are usually confined to a narrow stress range where a change in the stress sensitivity will be difficult to detect. Furthermore, if equation 10 describes the actual stress/creep-rate relation more adequately than equation 8, the change in stress sensitivity of the creep-rate with stress will be even more gradual than shown in fig. 2.

However, there is also a real effect that will tend to diminish the increasing stress sensitivity of the creep-rate with decreasing stress. In materials in which coalescence of the second-phase particles is not completely negligible, there will be time, at sufficiently small creep-rates, for some increase of the interparticle spacing due to coalescence. This decreases z in equation 8 somewhat, and therefore counteracts the increase of stress sensitivity. Hence, in such a case, the very high exponents close to the

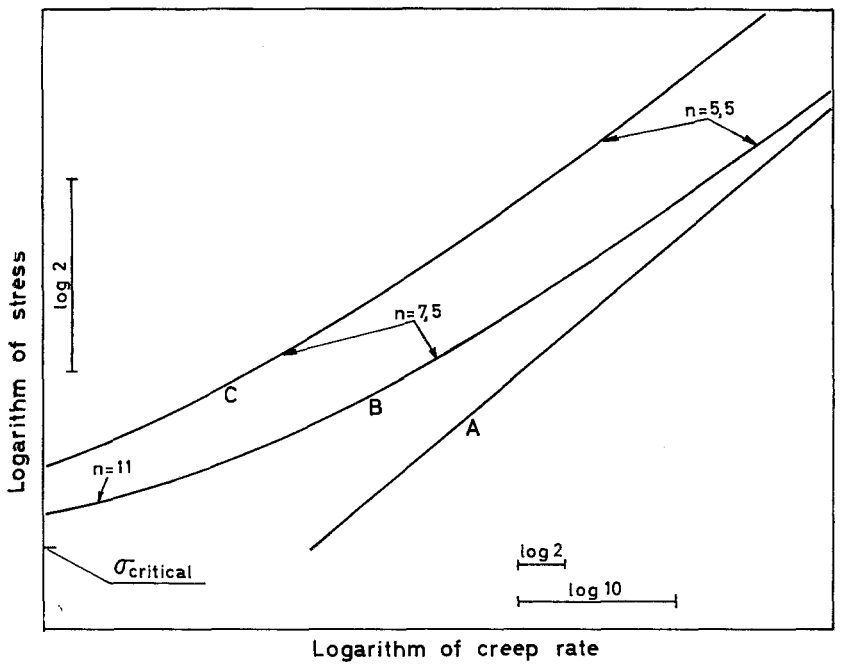

Figure 2 Schematic figure of the stress/creep-rate relation in a double logarithmic graph showing the effect of second-phase particles according to equation 8. Curve A represents a material free of particles, and C a material containing particles. For full explanation of the implication of the three curves A, B, and C, see text.

critical stress (fig. 2) will not be attained. In precipitation-hardened materials, in which coalescence cannot be entirely disregarded, it is thought that this will diminish the stress sensitivity at low creep-rates, even so much that no deflection as shown in fig. 2 occurs. However, in dispersion-hardened alloys, coalescence may be considered to be completely negligible because of the extremely low solubility of the oxide particles in the material, and the deflection in the stress/creep-rate curve shown in fig. 2 is therefore more likely to exist in these alloys. The observation by Ansell and Lenel [5] that the stress-dependence of the creep-rate in SAP increases with decreasing stresses supports these arguments.

It should also be borne in mind that at sufficiently high and low creep-rates other mechanisms, not directly depending on the existence of second-phase particles, may in certain cases come into play and eliminate the variable stress sensitivity. For instance, Sherby and Burke [18] state that the relation between stress and creep-rate for creep-rates below $\dot{\epsilon} = 10^9 D$ obeys a linear law, $\dot{\epsilon} \propto \sigma$, and for creep-rates above $\dot{\epsilon} = 10^9 D$ an exponential law, $\dot{\epsilon} \propto \exp$ (constant σ). Here D is the self-diffusion coefficient. The linear law is possibly related to diffusion creep and the exponential law to a high stress-induced vacancy production; both mechanisms may work equally well in materials with or without precipitates.

3. Summary and Conclusions

The current creep theories for alloys hardened by a second phase cannot rationalise certain features of the creep behaviour adequately. It is shown that a recovery creep model based upon the theory by McLean and co-workers [8-10] and modified with respect to the second-phase particles provides a satisfactory explanation of creep in these materials. In such a model the creep process is described as consecutive events of recovery and strain-hardening, where the recovery is the rate-controlling step. An addition of second-phase particles then reduces the creep-rate only through a decrease of the recovery rate. The recovery process consists of the growth of the meshes in the dislocation network. This is analogous to normal grain growth, and the impeding effect of particles on the latter can be used to explain the recovery of the dislocation network in the presence of second-phase particles. The particles retard the recovery partly through a decrease of the driving force

of the recovery process and partly through a decrease in the mobility of climbing dislocations.

It is shown that such a model is consistent with the large variations in the stress-dependence for the creep-rate that have been observed for dispersion- and precipitation-hardened alloys. It is also able to explain the increasing stress sensitivity with decreasing stress observed in SAP. The recovery creep model in materials hardened by a second phase is supported by recent results from creep in precipitation-hardened alloys, which showed that the creep-rate changed exclusively through a change in recovery rate when the state of hardening was altered.

Acknowledgements

The author thanks AB Atomenergi for permission to publish this paper and Dr G. Östberg for valuable discussions and continuous interest and support.

References

1. G. S. ANSELL and J. WEERTMAN, *Trans. Met. Soc. AIME* **215** (1959) 838.
2. B. A. WILCOX and A. H. CLAUER, *ibid* **236** (1966) 570.
3. B. A. WILCOX, A. H. CLAUER, and W. S. MCCAIN, *ibid* **239** (1967) 1791.
4. C. L. MEYERS, J. C. SHYNE, and O. D. SHERBY, *J. Austr. Inst. Met.* **8** (1963) 171.
5. G. S. ANSELL and F. V. LENEL, *Trans. Met. Soc. AIME* **221** (1961) 452.
6. W. BETTERIDGE, "The Nimonic Alloys" (Edward Arnold, London, 1959).
7. L. UNNEBERG, International report, AB Atomenergi, Stockholm, Sweden.
8. D. MCLEAN, *Reports Progr. Phys.* **29** (1966) 1.
9. D. MCLEAN and K. F. HALE, Structural Processes in Creep, Special Report No. 70 (Iron and Steel Institute, London, 1961) p. 19.
10. S. K. MITRA and D. MCLEAN, *Proc. Roy. Soc.* **295A** (1966) 288.
11. J. FRIEDEL, "Dislocations" (Pergamon Press, Oxford, 1964).
12. C. ZENER, *Trans.* AIME **175** (1949) 15.
13. M. HILLERT, *Acta Met.* **13** (1965) 227.
14. H. WIEDERSICH, *J. Metals* **16** (1964) 425.
15. F. GAROFALO, "Fundamentals of Creep and Creep-Rupture in Metals" (MacMillan, New York, 1965) p. 97.
16. T. B. GIBBONS, Swedish Institute of Metals Research Stockholm, Internal Report No. 64.
17. R. LAGNEBORG, to be published in *Met. Sci. J.*
18. O. D. SHERBY and P. M. BURKE, Dept. Materials Science, Stanford University, SU-DMS No. 67-33 (1967).

Addendum

If we assume already from the start of the analysis in section 2.3. that the $\sigma - \mu b\sqrt{\rho}$ relation has a stress intercept $\alpha\mu bz$, equations 6 to 8 and 10 will change slightly. The implication of equation 5, as used in this work, is that in the driving force factor $(\tau R_m/2)\,(1/R_m - z)^2$, values of R_m corresponding to the relation between σ and R_m with no friction stress $(\alpha\mu bz)$, $\sigma = \alpha\mu b \cdot 1/R_m$, should be used; the impeding effect of the second-phase particles on the rate of recovery is taken care of by the term z in the driving force. From the $\sigma - \mu b\sqrt{\rho}$ relation we obtain, $d\sigma/dt = (\alpha\mu b/R_m^2)\,(-dR_m/dt)$. In this expression R_m should be substituted by σ as given by $\sigma = \alpha\mu b\,(z + 1/R_m)$, i.e. the expression containing the stress intercept. Similarly, the introduction of the friction stress changes the strain hardening coefficient, $d\sigma/d\epsilon$, to be proportional to $1/(\sigma - \alpha\mu bz)$. Hence, the recognition of the friction stress, $\alpha\mu bz$, from the very beginning of the analysis leads to the following modifications of equations 6 to 8 and 10.

$$r = \frac{M\tau}{2} \cdot \frac{1}{\sigma}\left(\frac{\sigma}{\alpha\mu b} - z\right)^4 \qquad (6a)$$

$$r \propto \left(\frac{\sigma}{\alpha\mu b} - z\right)^4 \qquad (7a)$$

$$\dot{\epsilon} \propto \left(\frac{\sigma}{\alpha\mu b} - z\right)^5 \qquad (8a)$$

$$\dot{\epsilon} \propto \sigma\left(\frac{\sigma}{\alpha\mu b} - \frac{\sigma_0}{\alpha\mu b}\right)^4 \qquad (10a)$$

These modifications result in a more gradual change of the stress sensitivity of the creep-rate. None of the conclusions drawn on the present paper are, however, changed by these modifications.